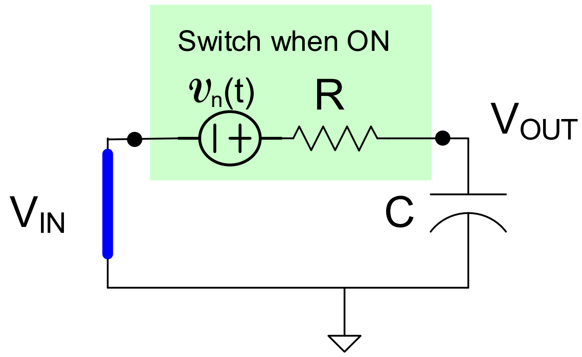
# EE 508
# Lecture 28

## Integrator Design

- Some other integrator structures
- Metrics for comparing integrators

Review from last lecture

# Noise during sampling phase




$$v_{n_{RMS}} = \sqrt{\frac{kT}{C}}$$

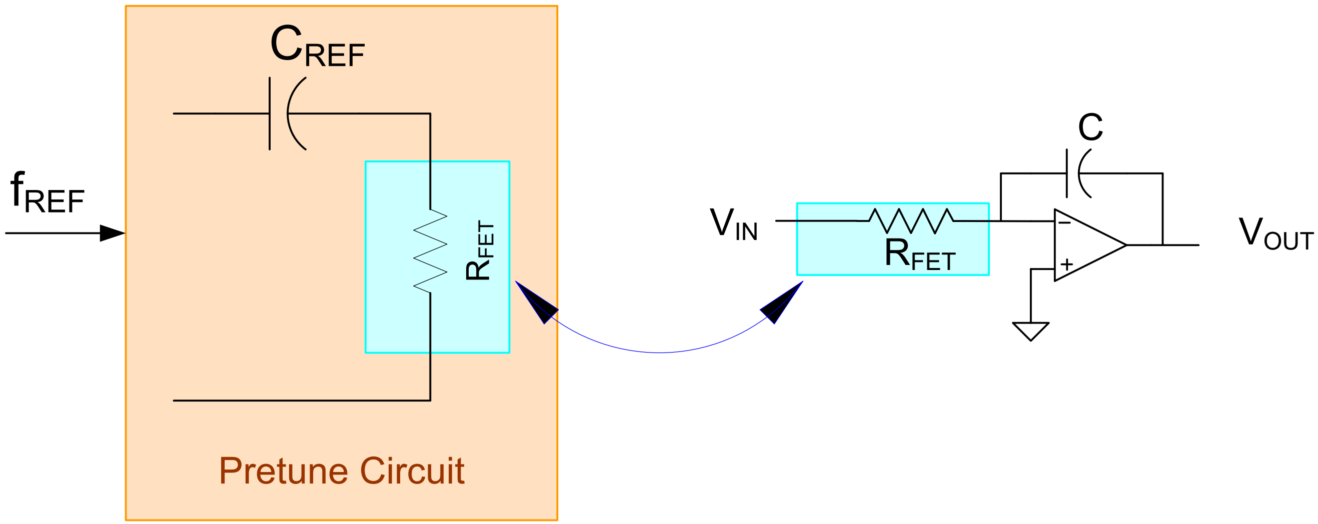
RMS noise voltage on C is independent of the state of the switch

So sampled RMS noise voltage should be same as instantaneous RMS voltage

Highly temperature dependent

Review from last lecture

# Switched-Resistor Voltage Mode Integrators

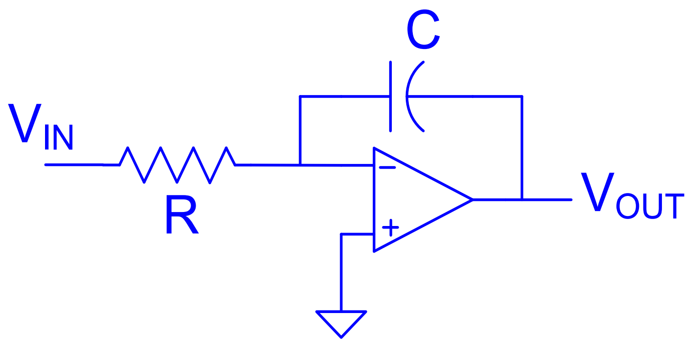



Switched-resistor integrator

- Accurate $CR_{FET}$ products is possible
- Area reduced compared to Active RC structure because $R_{FET}$ small
- Single pretune circuit can be used to “calibrate” large number of resistors
- Clock frequency not fast and not critical (but accuracy of $f_{REF}$ is important)
- Since resistors are memoryless elements, no transients associated with switching
- Since filter is a feedback structure, speed limited by BW of op amp

Review from last lecture

# Variants of basic inverting integrator have been considered




Basic Miller Integrator

- Active RC
- MOSFET-C
- OTA-C
- $g_m$-C
- Switched-Capacitor
- Switched-Resistor

**Performance of all is limited by GB of Operational Amplifiers**

How can integrator performance be improved?

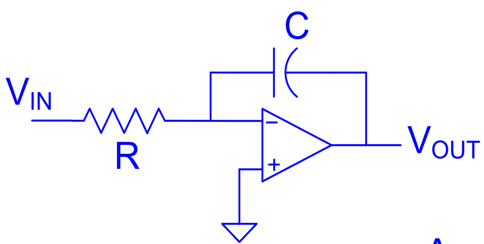
- Better op amps
- Better Integrator Architectures

How can the performance of integrator structures be compared?

Need metric for comparing integrator performance

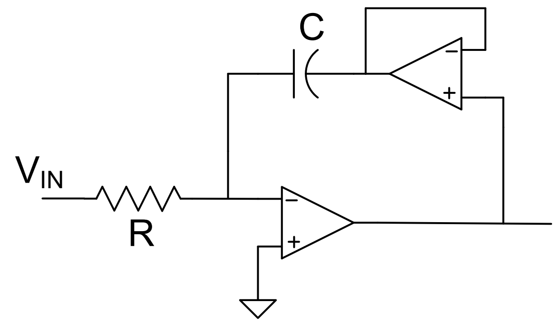
Review from last lecture

# Are there other integrators in the basic classes that have been considered?



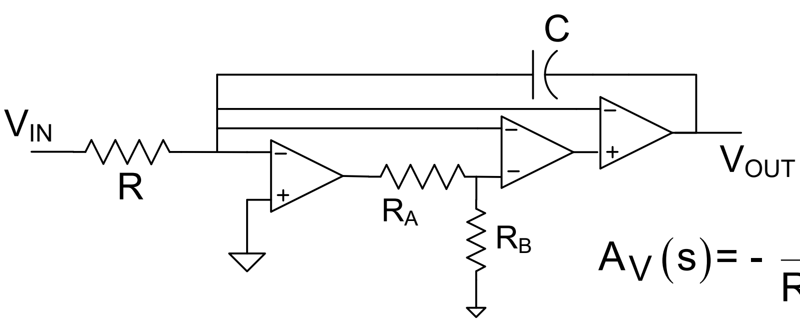

$$A_V(s) = -\frac{1}{RCs}$$

Miller Inverting



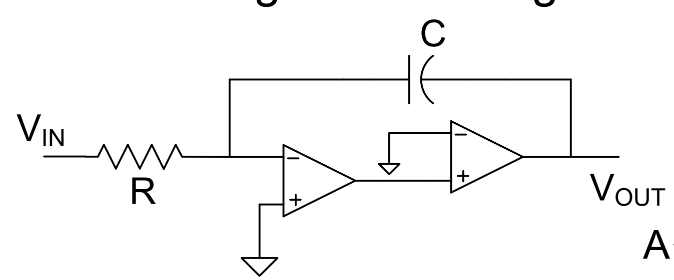

$$A_V(s) = -\frac{1}{RCs}$$

High-Q Inverting



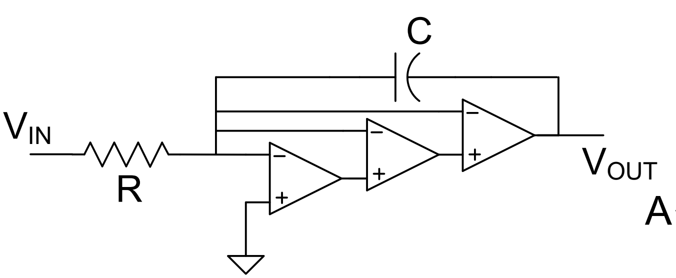

$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



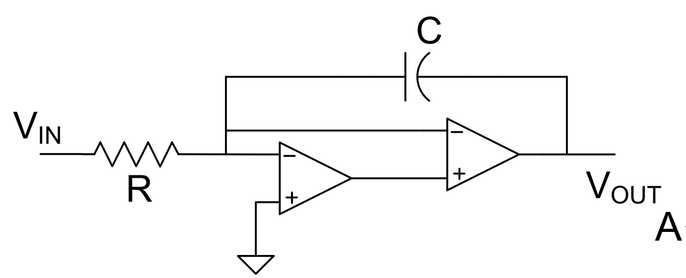

$$A_V(s) = -\frac{1}{RCs}$$

Cascaded Inverting




$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting

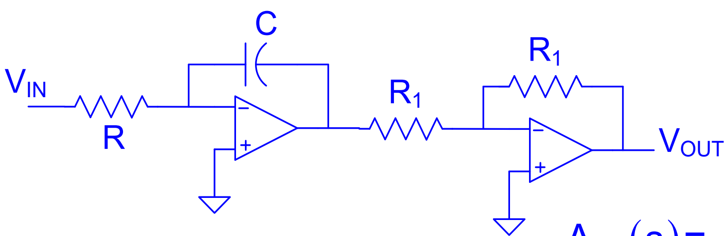



$$A_V(s) = -\frac{1}{RCs}$$

Zero Sensitivity Inverting

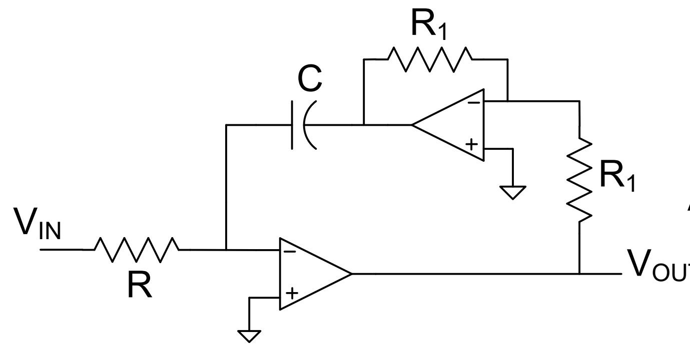
Review from last lecture

# Are there other integrators in the basic classes that have been considered?



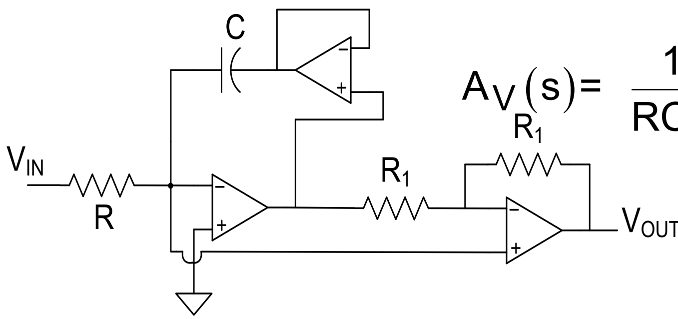

$$A_V(s)=\frac{1}{RCs}$$

Miller Noninverting



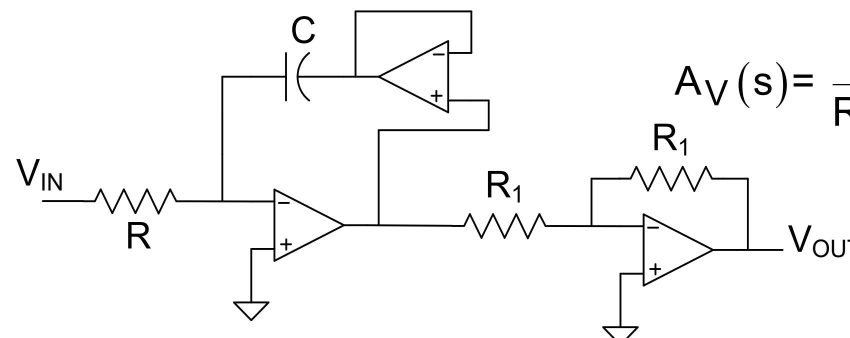

$$A_V(s)=\frac{1}{RCs}$$

Phase Lead Integrator



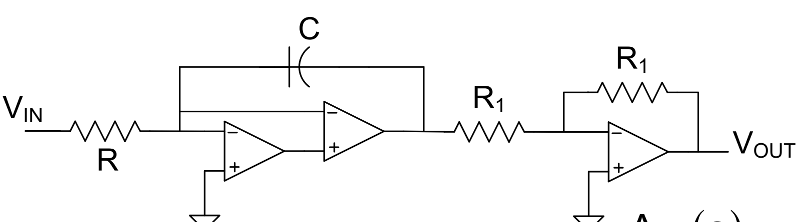

$$A_V(s)=\frac{1}{RCs}$$

Modified High-Q Noninverting



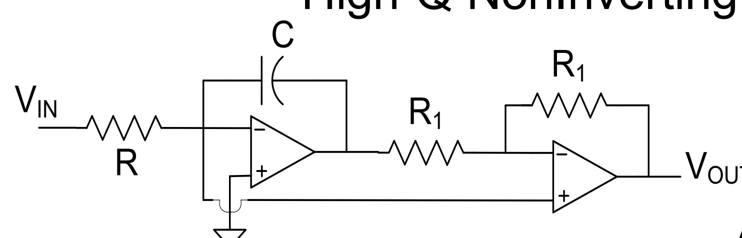

$$A_V(s)=\frac{1}{RCs}$$

High-Q NonInverting




$$A_V(s)=\frac{1}{RCs}$$

Zero Sensitivity Noninverting

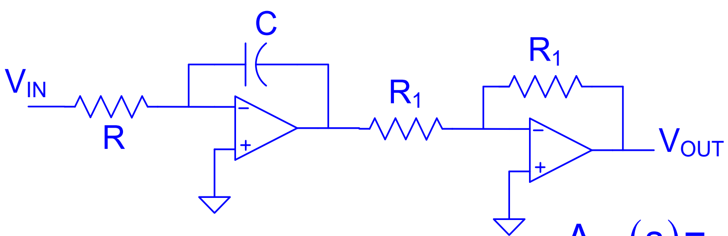



$$A_V(s)=\frac{1}{RCs}$$

Modified Miller Noninverting

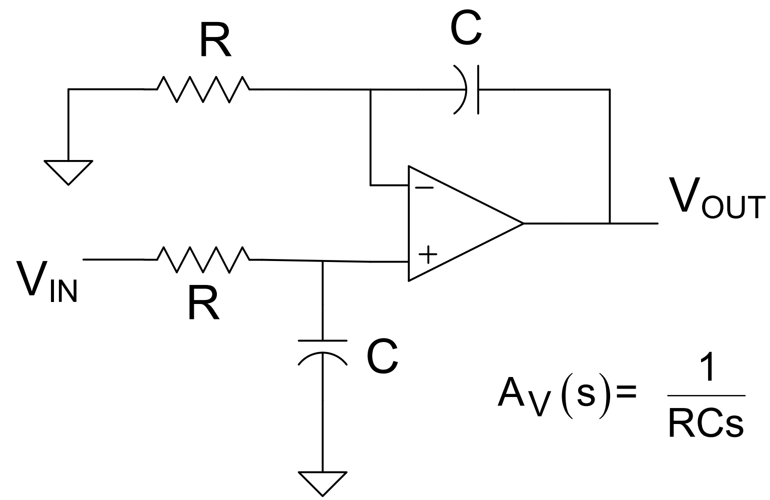
Review from last lecture

# Are there other integrators in the basic classes that have been considered?



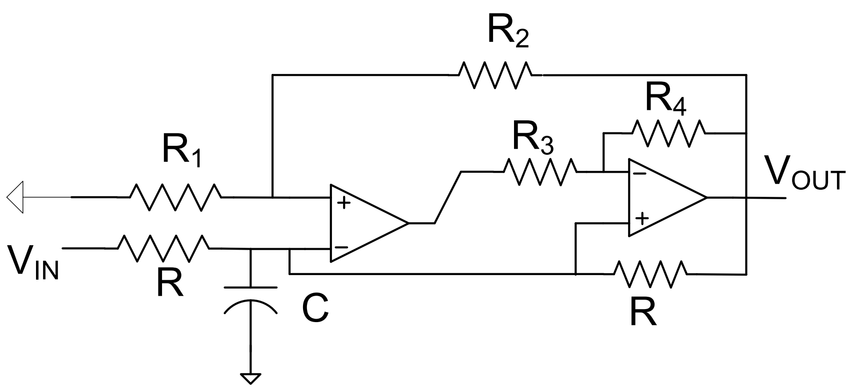

Miller Noninverting

$$A_V(s) = \frac{1}{RCs}$$




$$A_V(s) = \frac{1}{RCs}$$

Balanced Time Constant Noninverting




Zero Sensitivity Noninverting

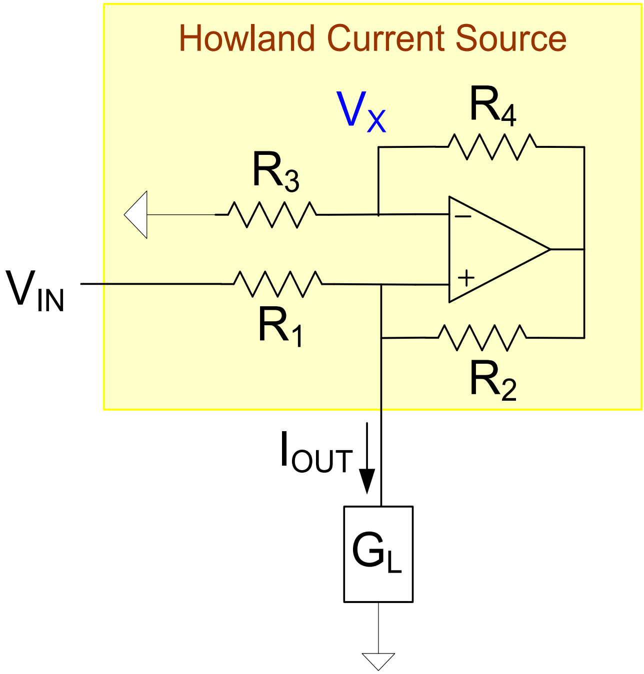
$$A_V(s) = \frac{2}{RCs}$$

If $R_1=R_2$ and $R_3=R_4$

(note this has a grounded integrating capacitor!)

# De Boo Integrator

Consider the Howland Current Source




$$I_{OUT} = V_{IN}G_1 + \left[ V_X \left( \frac{G_2G_3}{G_4} - G_1 \right) \right]$$

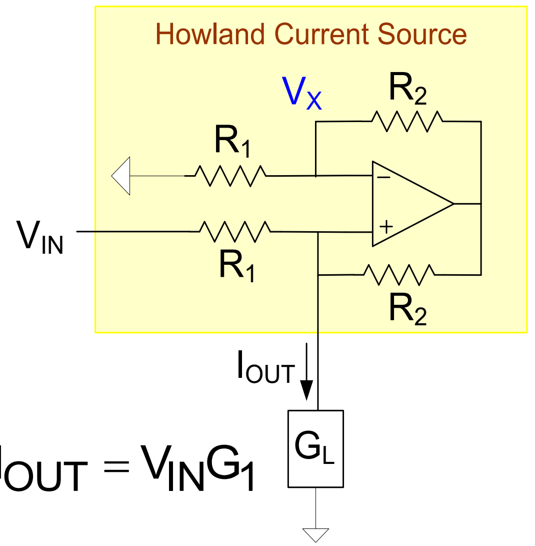
If resistors sized so that $G_1 = \frac{G_2G_3}{G_4}$

$$I_{OUT} = V_{IN}G_1$$

Since $I_{OUT}$ is independent of $V_X$, behaves as an ideal current source!

If sizing constraints are satisfied, behaves as a constant-current source that can drive a grounded load

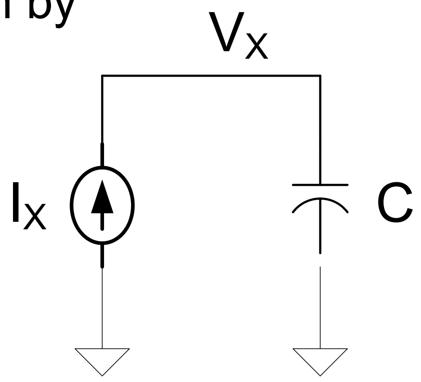
# DeBoo Integrator




$$I_{OUT} = V_{IN} G_1$$

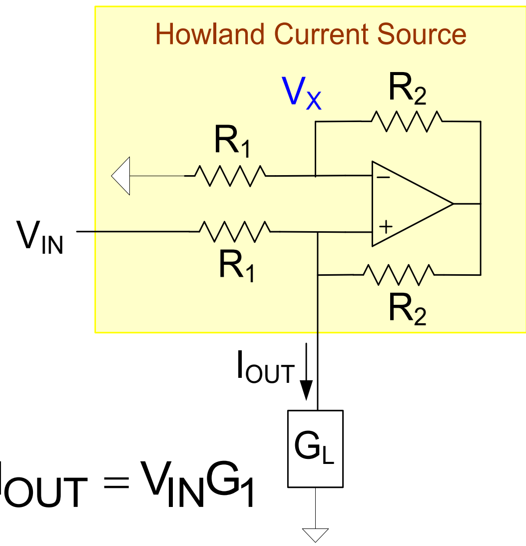
Observe that if a current source drives a grounded capacitor, then the nodal voltage on the capacitor is given by

$$V_X = I_X \frac{1}{sC}$$

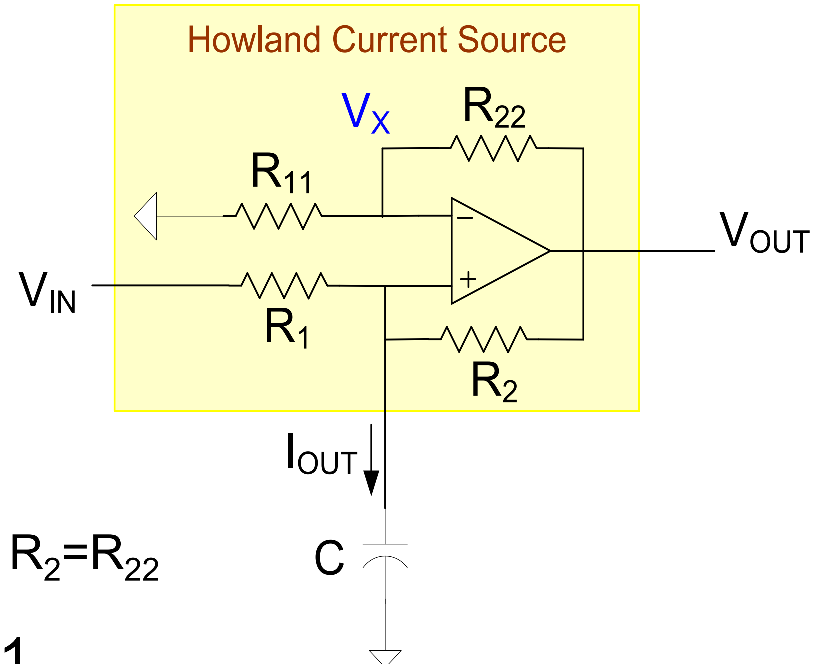



Thus, if $I_X$ is proportional to $V_{IN}$, the voltage on the capacitor would be a weighted integral of $V_{IN}$

# De Boo Integrator




$$I_{OUT} = V_{IN} G_1$$



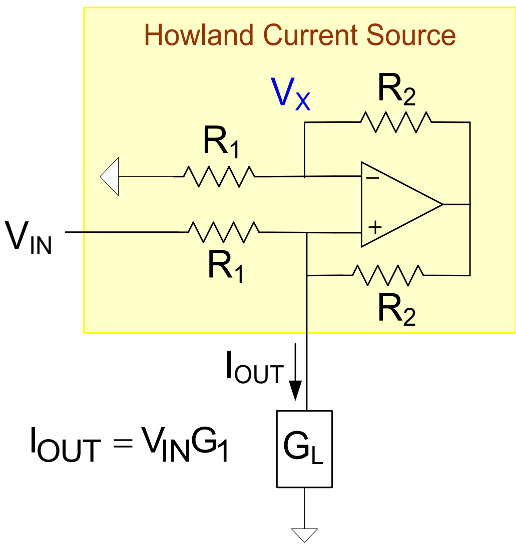

If $R_1 = R_{11}$ and $R_2 = R_{22}$

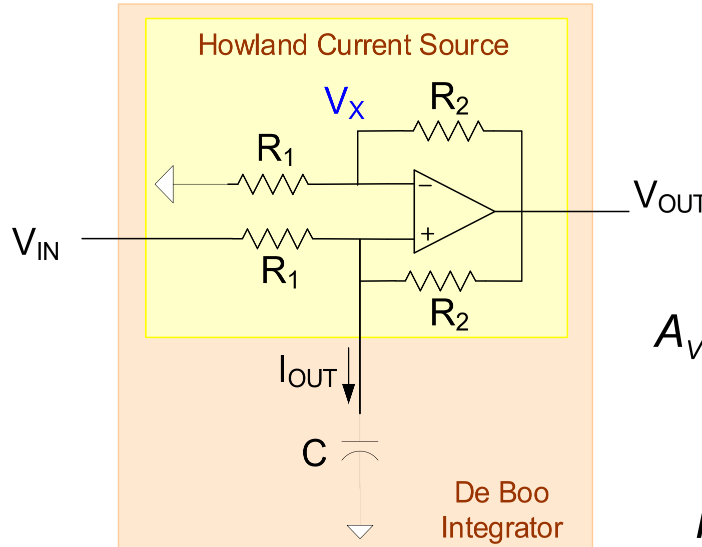
$$V_X = \frac{V_{IN}}{R_1} \frac{1}{sC}$$

$$V_{OUT} = V_X \left(1 + \frac{R_{22}}{R_{11}}\right) = \frac{V_{IN}}{R_1} \frac{1}{sC} \left(1 + \frac{R_{22}}{R_{11}}\right)$$

$$A_V(s) = \frac{1}{s} \left[ \frac{1}{R_1 C} \left(1 + \frac{R_{22}}{R_{11}}\right) \right]$$

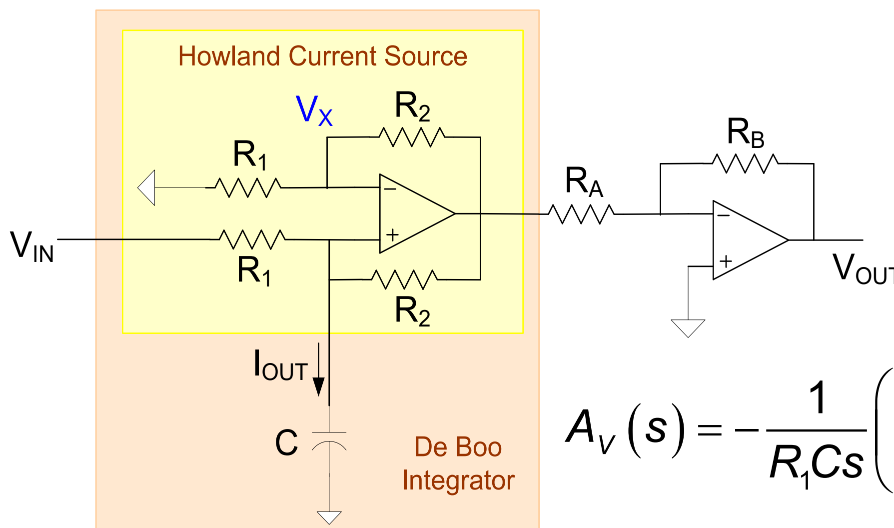
# De Boo Integrator




$$A_V(s) = \frac{1}{R_1 C s}\left(1 + \frac{R_2}{R_1}\right)$$

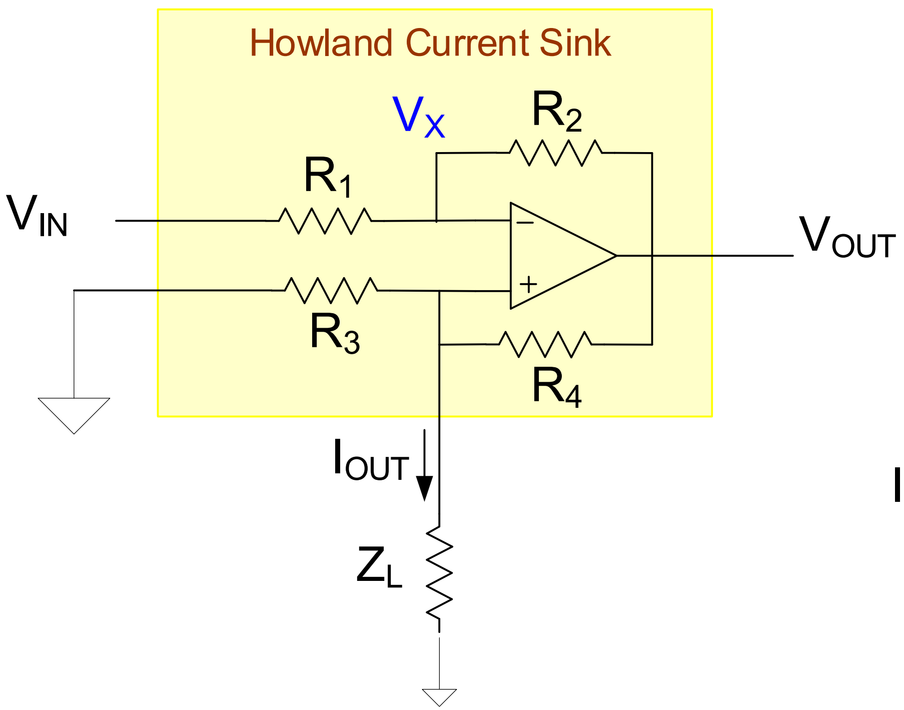
$$I_O(s) = \frac{1}{R_1 C}\left(1 + \frac{R_2}{R_1}\right)$$




$$A_V(s) = -\frac{1}{R_1 C s}\left(1 + \frac{R_2}{R_1}\right)\frac{R_B}{R_A}$$

# De Boo Integrator

Consider the Sinking Howland Current Source




$$\left.\begin{array}{l} I_{OUT} = (V_{OUT} - V_X)G_4 - G_3 V_X \\ V_X(G_1+G_2) = G_1 V_{IN} + G_2 V_{OUT} \end{array}\right\}$$

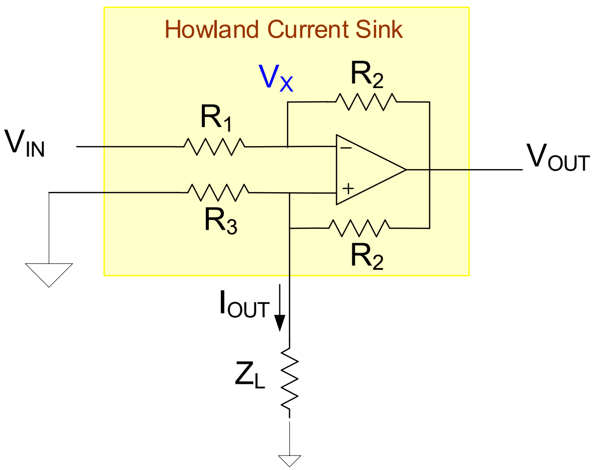
$$I_{OUT} = -\frac{G_3+G_4}{G_1+G_2} G_1 V_{IN} + \left[\frac{G_1G_4 - G_2G_3}{G_1+G_2}\right] V_{OUT}$$

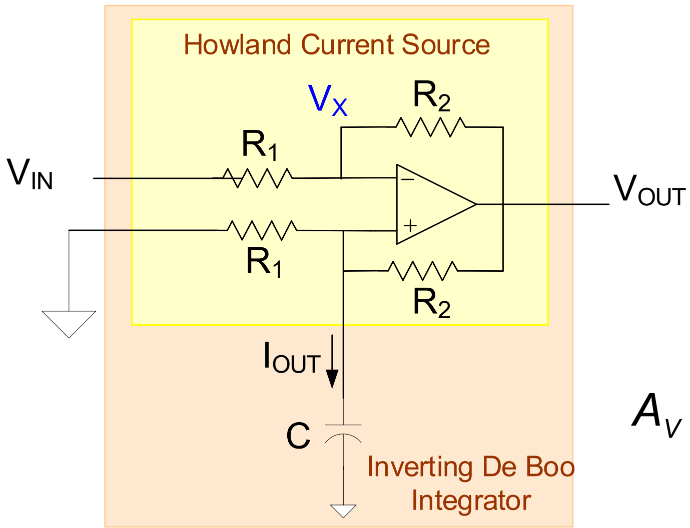
If resistors sized so that $G_1 = \frac{G_2G_3}{G_4}$

$$I_{OUT} = -G_1 V_{IN}$$

If sizing constraints are satisfied, behaves as a sinking constant-current source that can drive a grounded load

# De Boo Integrator




$$A_V(s) = -\frac{1}{R_1Cs}\left(1+\frac{R_2}{R_1}\right)$$

$$I_O(s) = -\frac{1}{R_1C}\left(1+\frac{R_2}{R_1}\right)$$

$$I_{OUT} = -G_1V_{IN}$$

# Observations:

Many different integrator architectures ideally provide the same gain

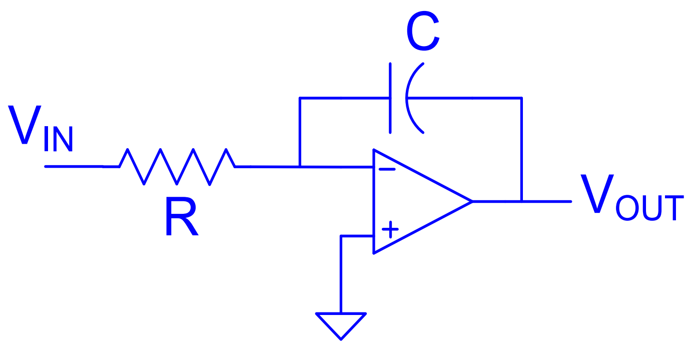
Similar observations can be made for other classes of integrators

Are there benefits or limitations for using the different integrator structures?

How can the performance of an integrator be characterized and how can integrators be compared?

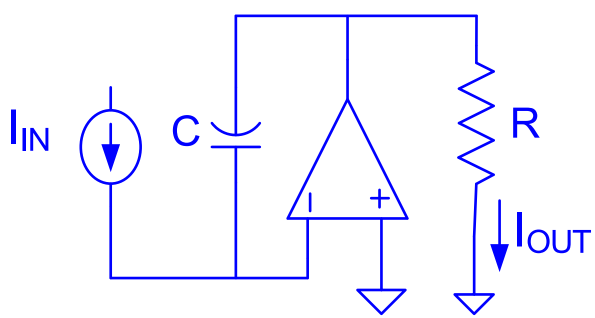
# Before considering performance of the integrators consider current-mode integrators

Basic Miller Integrators




$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{RCs}$$

Voltage Mode

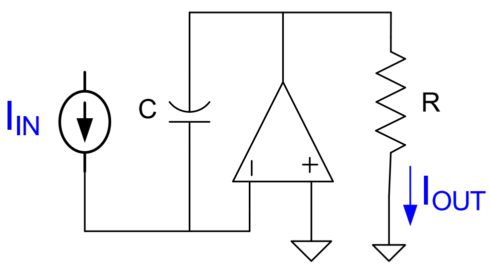



$$\frac{I_{OUT}}{I_{IN}} = -\frac{1}{RCs}$$

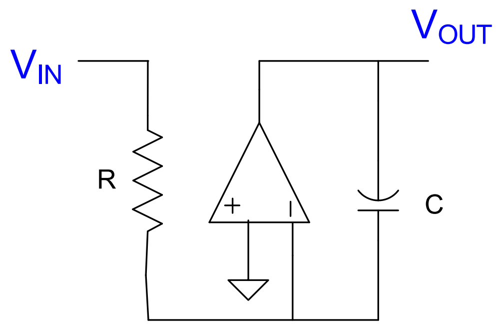
Current Mode

- Both use the same number of components and same type of components
- Many authors claim the current-mode integrator offers several major benefits to the voltage-mode integrator
- Will discuss the differences in more detail later

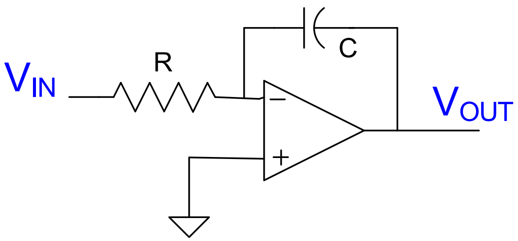
# Basic Miller Integrators




$$\frac{I_{OUT}}{I_{IN}} = -\frac{1}{RCs}$$




$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{RCs}$$

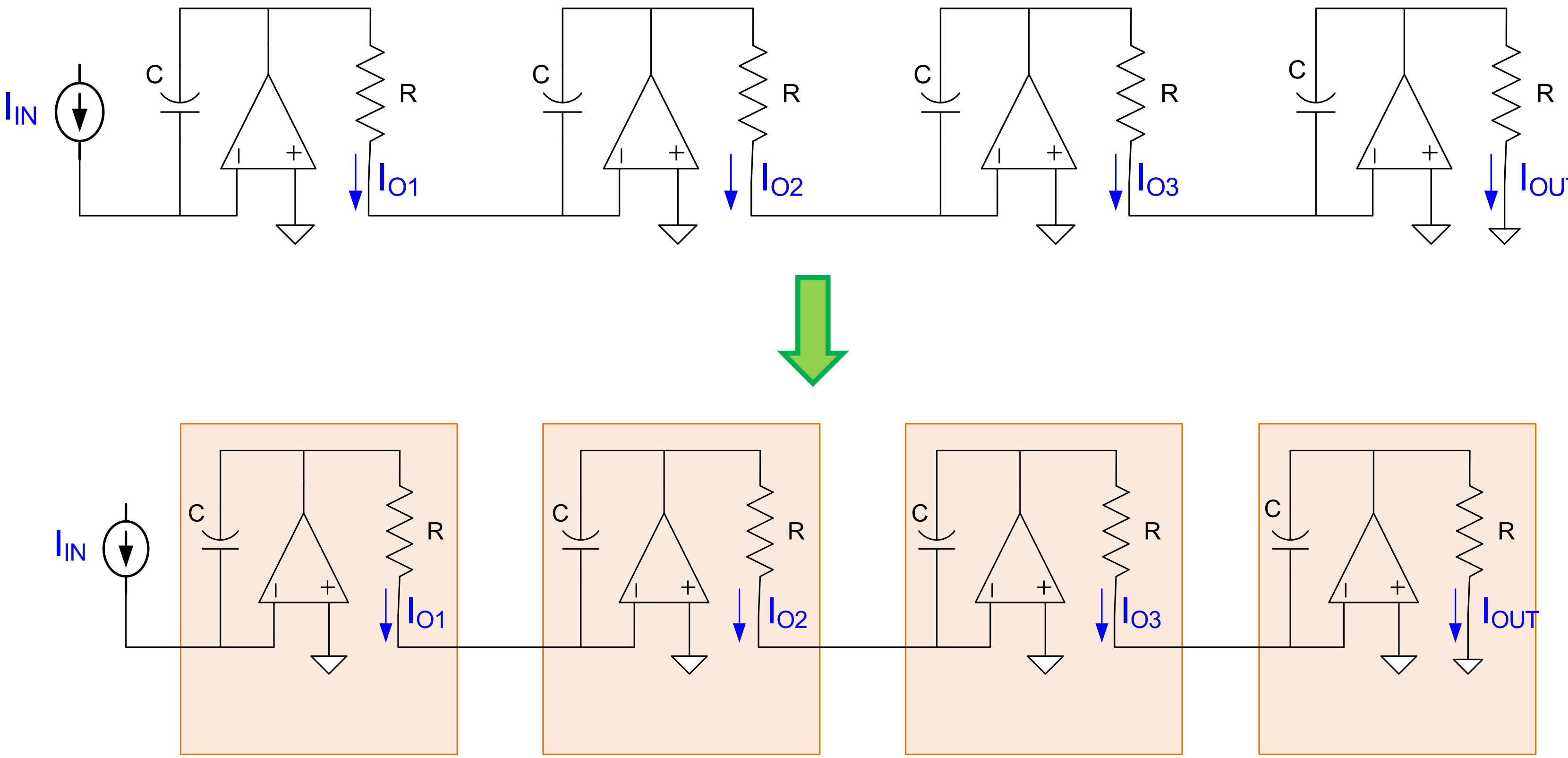



$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{RCs}$$

How does the one in the middle differ from the one on the right?

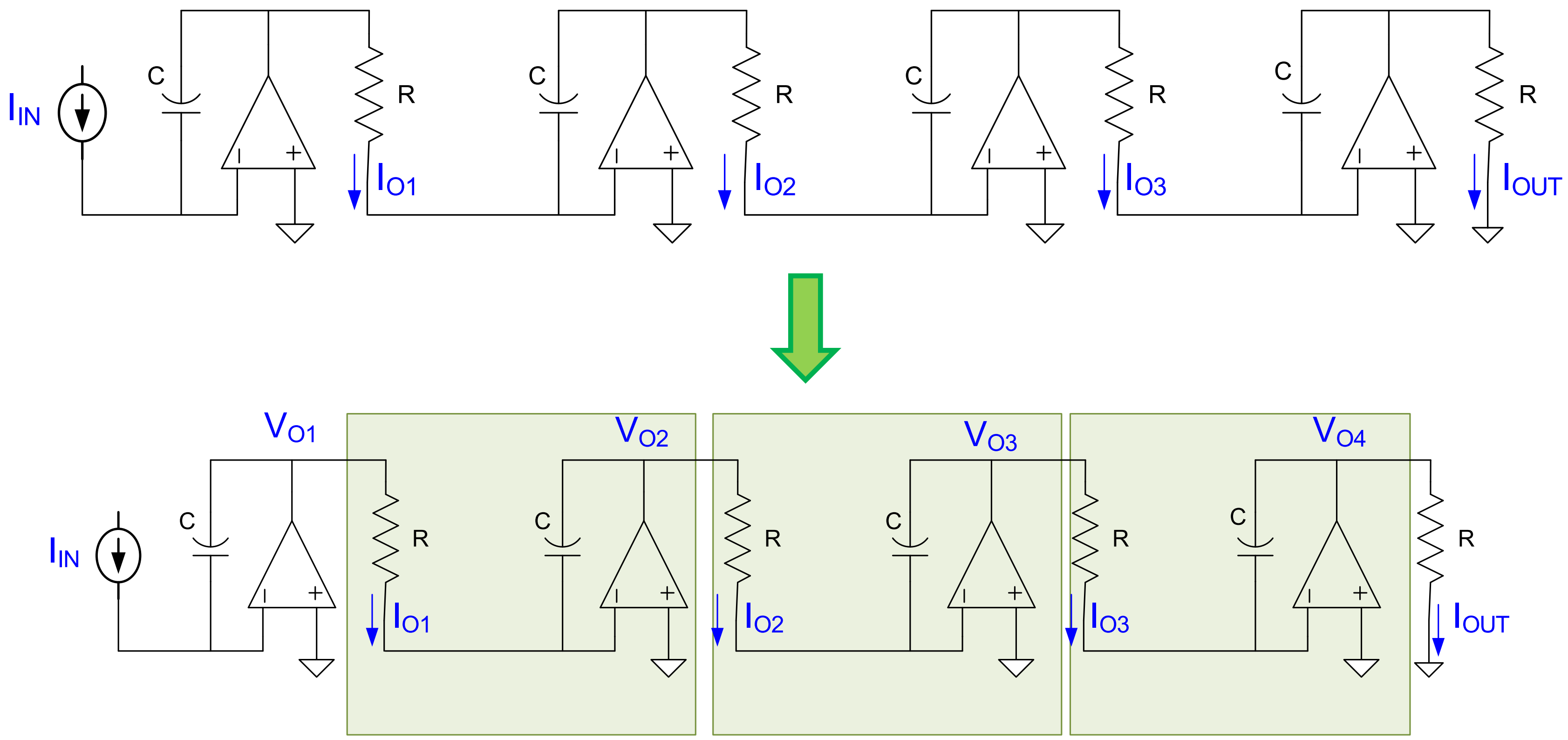
# Preliminary Comparison of Current Mode Integrators with Voltage Mode Integrators

Consider Integrator Cascade



# Preliminary Comparison of Current Mode Integrators with Voltage Mode Integrators

Consider Integrator Cascade



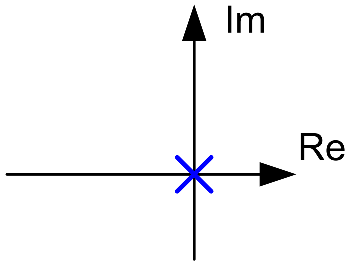

If connected in a loop, structures are identical !

# How can the performance of an integrator be characterized and how can integrators be compared?

Consider Ideal Integrator Gain Function

$$A_V(s)=\frac{I_0}{s} \qquad A_V(j\omega)=\frac{I_0}{j\omega}$$

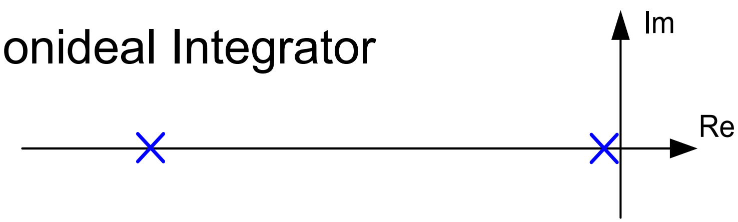
Ideal Integrator




Consider a nonideal integrator Gain Function

$$A_V(s)=\frac{\alpha I_{01}}{s+\alpha}A_{OO}(s)$$

Nonideal Integrator




Key characteristics of an ideal integrator:

- Magnitude of the gain at $I_0$=1
- Phase of integrator always $90^o$
- Gain decreases with $1/\omega$

Are any of these properties more critical than others?

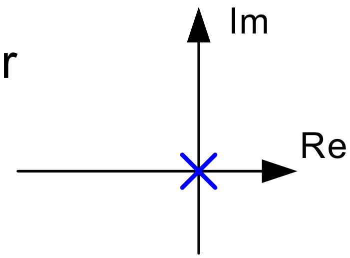
In many applications:

**Key property of ideal integrator is a phase shift of $90^o$ at frequencies around $I_0$!**

# How can the performance of an integrator be characterized and how can integrators be compared?

**Is stability of an integrator of concern?**

Ideal Integrator




- Ideal integrator is not stable
- Integrator function is inherently ill-conditioned
- Integrator is almost never used open-loop
- Stability of integrator not of concern, stability of filter using integrator is of concern
- Some integrators may cause unstable filters, others may result in stable filters
- Instability in filter because desired poles move in RHP is of little concern since the filter performance would be unacceptable long before the stability became an issue
- Instability in filter due to parasitic poles is of concern but not a problem in most circuits

# How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$A_V(j\omega)=\frac{\pm 1}{R(\omega)+jX(\omega)}$$

where $R(\omega)$ and $X(\omega)$ are real and represent the real and imaginary parts of the denominator respectively

$$\text{Phase}=-\tan^{-1}\left(\frac{X(\omega)}{R(\omega)}\right)$$

Ideally $R(\omega)=0$

Definition: The Integrator Q factor is the ratio of the imaginary part of the denominator to the real part of the denominator

$$Q_{INT}=\left(\frac{X(\omega)}{R(\omega)}\right)$$

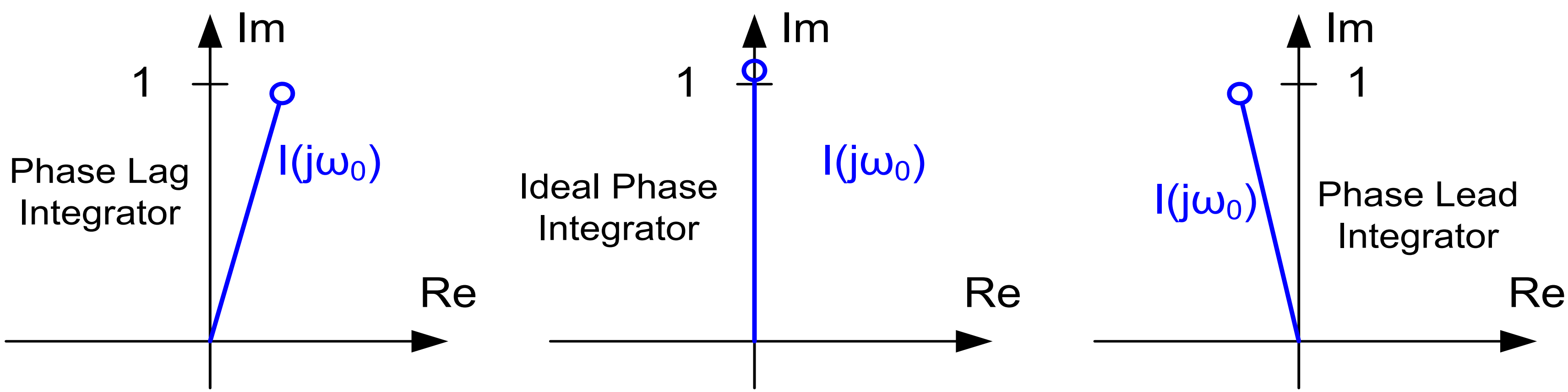
Typically most interested in $Q_{INT}$ at the nominal unity gain frequency of the integrator

# How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$I_V(j\omega)=\frac{\pm 1}{R(\omega)+jX(\omega)}$$

Lead/Lag Characteristics for <u>Inverting</u> Integrators (inverting gain at $\omega_0$ should be 1 at angle of 90°)




$\omega_0$ is the unity gain frequency of the integrator

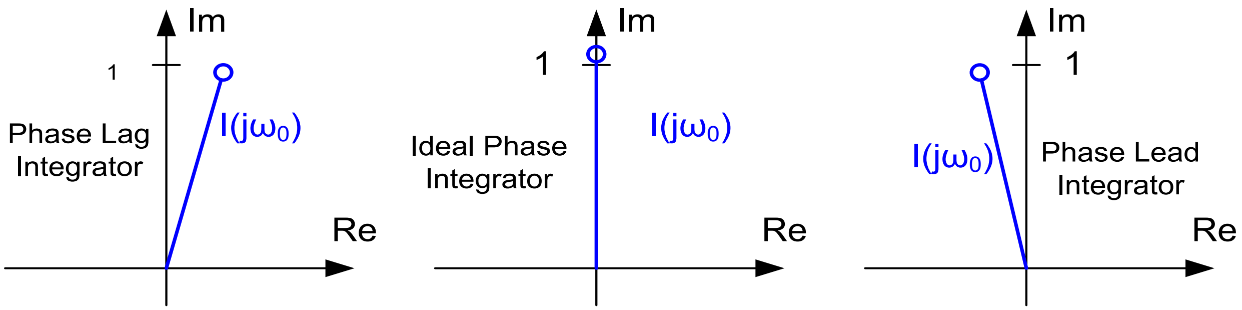
For Phase Lag Integrators, $R(\omega)$ is negative

For Phase Lead integrators, $R(\omega)$ is positive

# How can the performance of an integrator be characterized and how can integrators be compared?

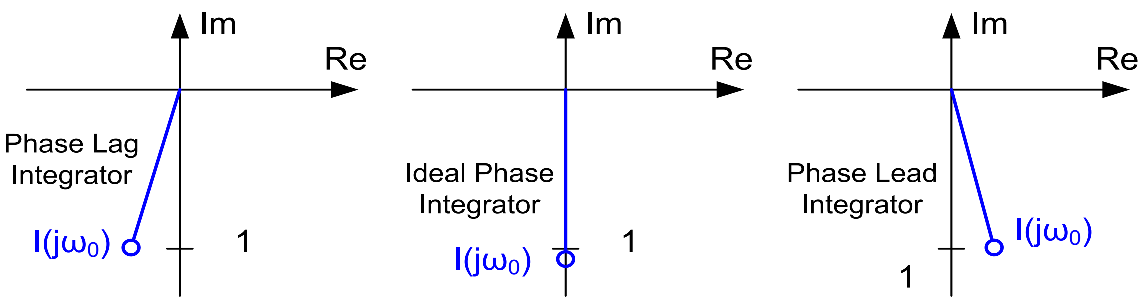
Lead/Lag Characteristics for Inverting Integrators

$$I_V(j\omega)=\frac{-1}{R(\omega)+jX(\omega)}$$




For Phase Lag Integrators, $R(\omega)$ and $X(\omega)$ have opposite signs. For Phase Lead integrators, $R(\omega)$ and $X(\omega)$ have the same sign. Phase shift ideally 90° (actually -270°)

Lead/Lag Characteristics for Noninverting Integrators (noninverting gain at $\omega_0$ should be 1 at angle of -90°)

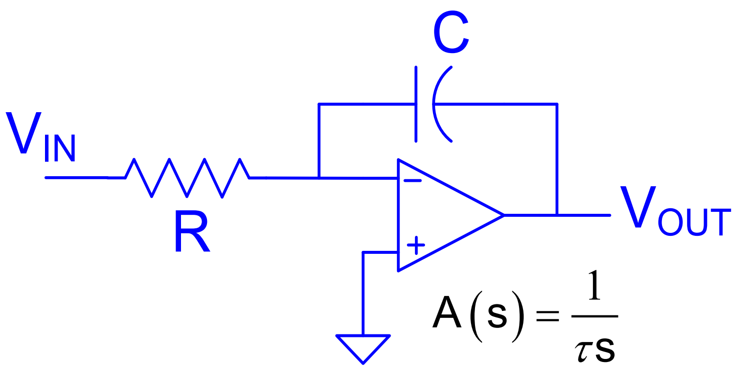



$$I_V(j\omega)=\frac{1}{R(\omega)+jX(\omega)}$$

For Phase Lag Integrators, $R(\omega)$ and $X(\omega)$ have opposite signs. For Phase Lead integrators, $R(\omega)$ and $X(\omega)$ have the same sign. Phase shift ideally 270°

# Integrator Q Factor

Consider Miller Inverting Integrator




$$A(s) = \frac{1}{\tau s}$$

$$A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)}$$

$$A_V(s) = \frac{-1}{RCj\omega + \tau j\omega(1 + RCj\omega)}$$

$$A_V(s) = \frac{-1}{-\tau\omega^2 RC + j(\omega[RC + \tau])}$$

Normalizing by $\omega_n = \omega RC$ and $\tau_n = \tau/RC = I_{0n}/GB$

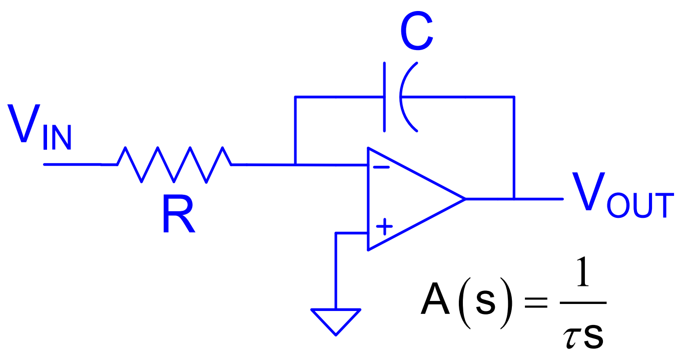
$$A_V(s) = \frac{-1}{-\tau_n \omega_n^2 + j(\omega_n[1 + \tau_n])}$$

Observe this integrator has excess phase shift (more than 90º in the denominator) at all frequencies

# Integrator Q Factor

$$A_V(j\omega)=\frac{\pm 1}{R(\omega)+jX(\omega)} \qquad Q_{INT}=\left(\frac{X(\omega)}{R(\omega)}\right)$$

Consider Miller Inverting Integrator



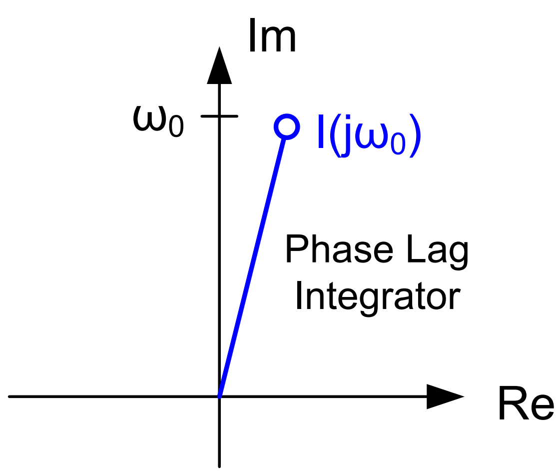

$$A(s)=\frac{1}{\tau s}$$

$$A_V(s)=\frac{-1}{-\tau_n\omega_n^2+j\left(\omega_n[1+\tau_n]\right)}$$

$$Q_{INT}=-\frac{\omega_n[1+\tau_n]}{\tau_n\omega_n^2}\cong-\frac{1}{\tau_n\omega_n}$$

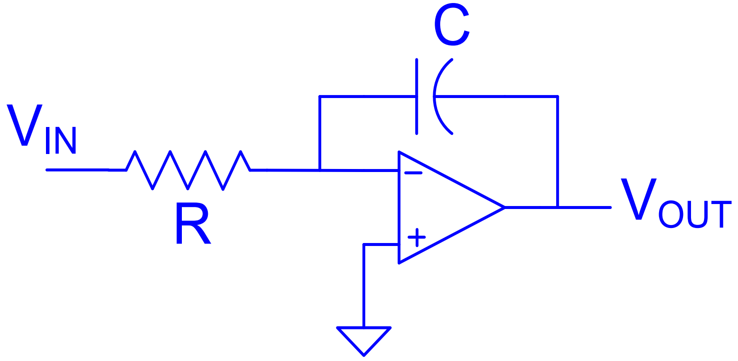
$$Q_{INT}\Big|_{\omega_N=1}\cong-\frac{1}{\tau_n\omega_n}\Big|_{\omega_N=1}\cong-\frac{1}{\tau_n}\Big|=-\frac{GB}{\omega_0}=-A(I_{0N})=-A$$

Since the phase is less than 90°, the Miller Inverting Integrator is a Phase Lag Integrator and $Q_{INT}$ is negative



# Integrator Pole Locations

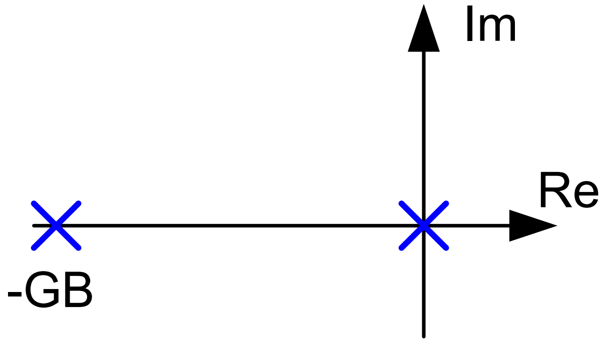
Consider Miller Inverting Integrator



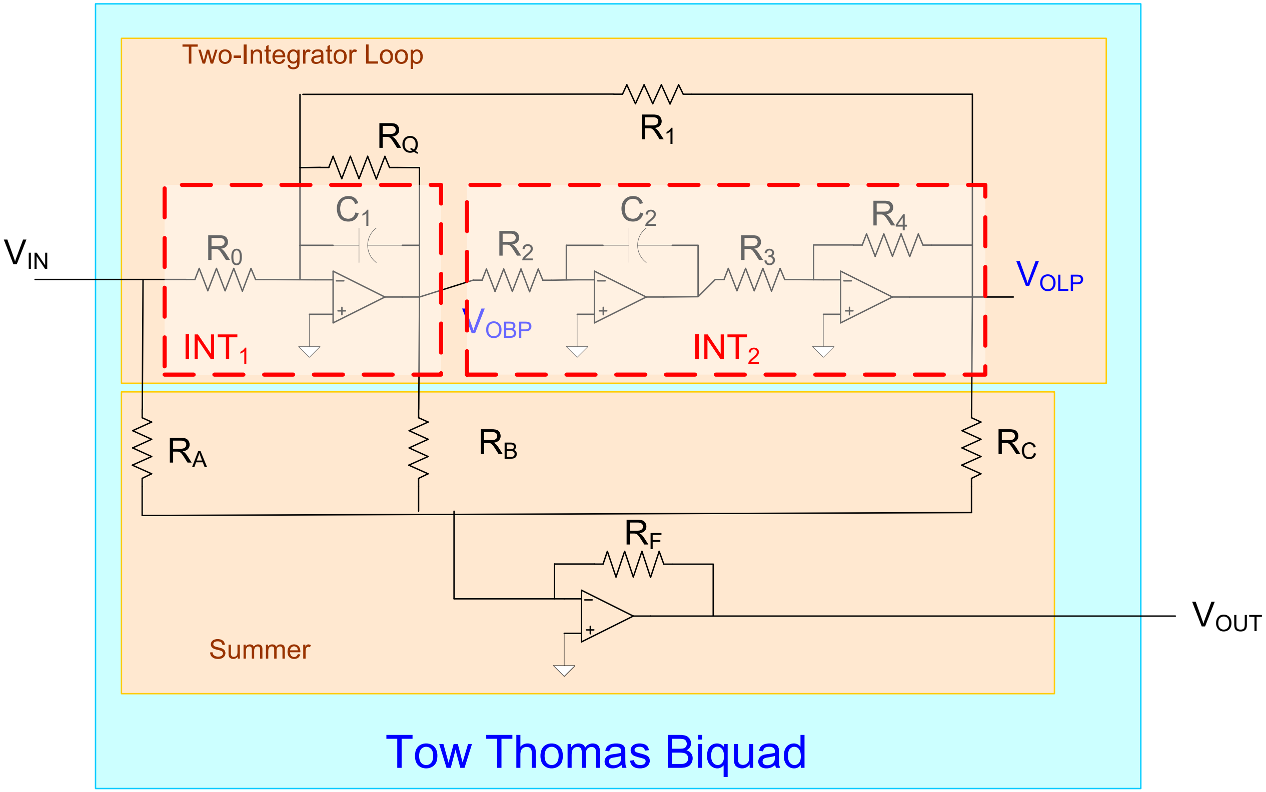

$$A_V(s) = \frac{-1}{RCs + \tau s(1+RCs)}$$

$$I_0 = 1/(RC)$$

Poles at s=0 and $s = -I_0(1+GB/I_0) \simeq -GB$



# Is the integrator Q factors simply a metric or does it have some other significance?

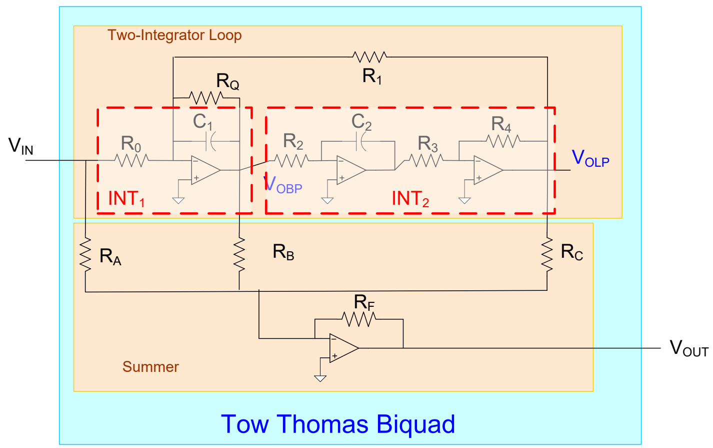



It can be shown that the pole Q for the TT Biquad can be approximated by

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}}$$

where $Q_{INT1}$ and $Q_{INT2}$ are evaluated at $\omega=\omega_o$

# Is the integrator Q factors simply a  metric or does it have some other significance?




It can be shown that the pole Q for the TT Biquad can be approximated by

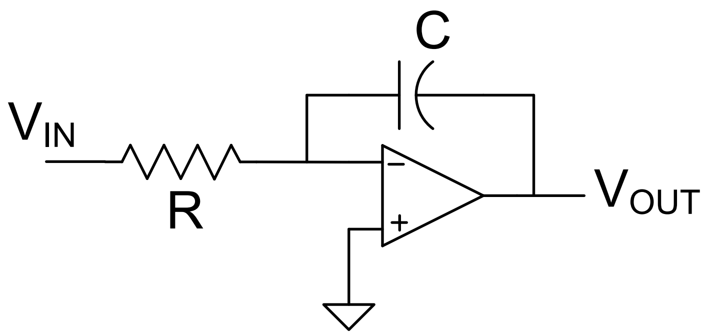
$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}(\omega_0)} + \frac{1}{Q_{INT2}(\omega_0)}}$$

Similar expressions for other second-order biquads

Observe that the integrator Q factors adversely affect the pole Q of the filter

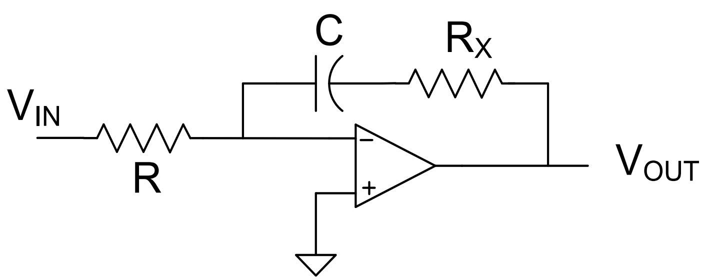
Observe that if $Q_{INT1}$ and $Q_{INT2}$ are of opposite signs and equal magnitudes, nonideal effects of integrator can cancel

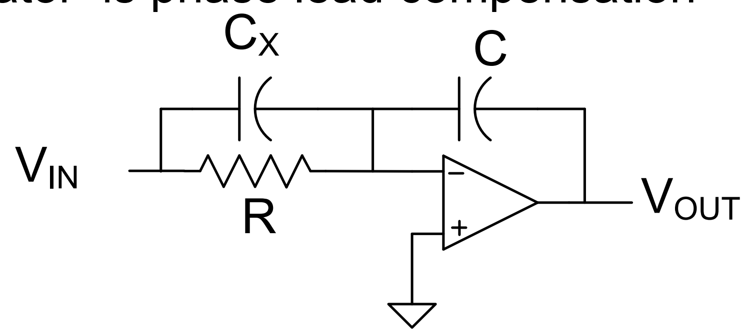
# What can be done to correct the phase problems of an integrator?




$$A_V(s)=\frac{-1}{RCs+\tau s(1+RCs)}$$

One thing that can help the Miller Integrator is phase-lead compensation




$$A_V(s)=\frac{-(1+R_XCs)}{RCs+\tau s(1+[R+R_X]Cs)}$$

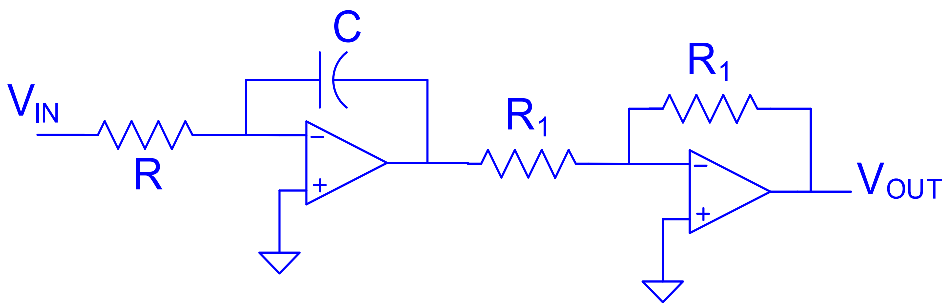
$$A_V(s)=\frac{-(1+RC_Xs)}{RCs+\tau s(1+R[C+C_X]s)}$$

$R_x$ and $C_x$ will add phase-lead by introduction of a zero

$R_X$ and $C_X$ will be small components

# Integrator Q Factor

Consider Miller Noninverting Integrator




$$A_V(s)=\frac{-1}{RCs+\tau s(1+RCs)}\bullet\frac{-1}{1+2\tau s}$$

$$A_V(j\omega)=\frac{1}{-3\tau RC\omega^2+j[\omega RC(1+2\tau\omega)]}$$

$$A_V(j\omega)\cong\frac{1}{-3\tau RC\omega^2+j\omega RC}$$

Observe this integrator has excess phase shift (more than 90° in the denominator) at all frequencies

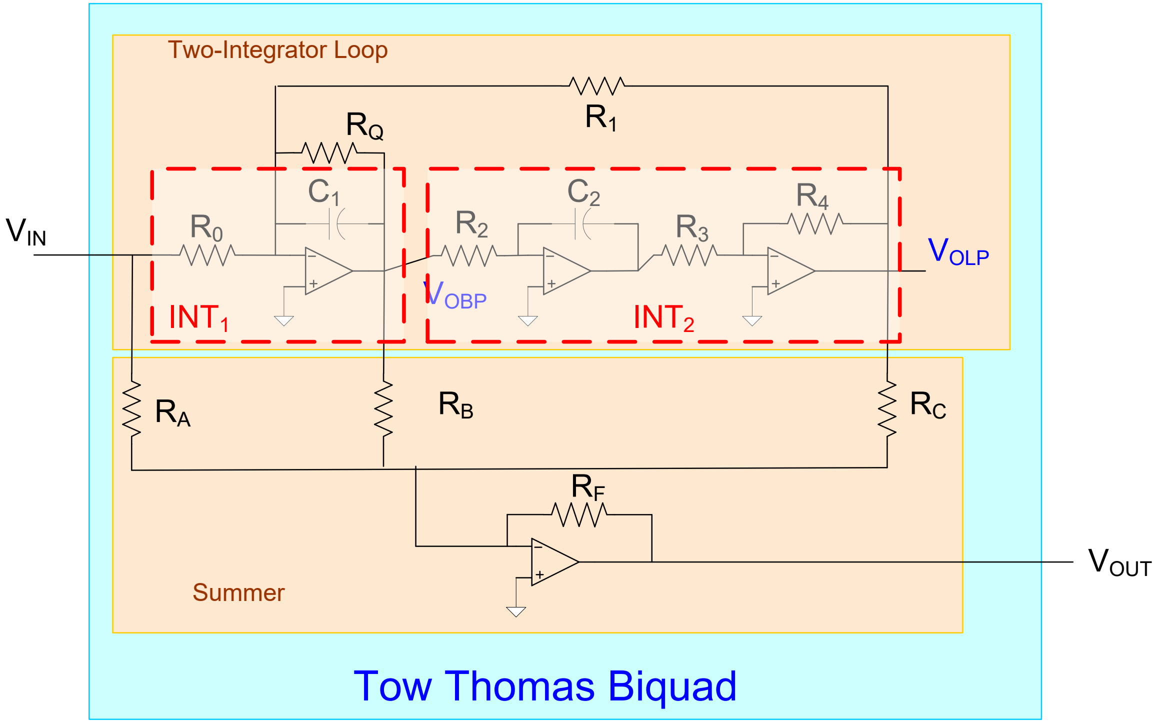
$$Q_{INT}\cong\frac{\omega RC}{-3\tau RC\omega^2}=\frac{-1}{3\tau\omega}$$

$$Q_{INT}\cong\frac{-1}{3}\left(\frac{GB}{\omega}\right)=-\frac{1}{3}\left|A(j\omega)\right|$$

Note: The Miller Noninverting Integrator has a modestly poorer $Q_{INT}$ than the Miller Inverting Integrator

# Example:

If $f_0$=10KHz, GB=1MHz, $Q_{NOM}$=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.




$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}}$$

# Example:

If $f_0$=10KHz, GB=1MHz, $Q_{NOM}$=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}}+\frac{1}{Q_{INT1}}+\frac{1}{Q_{INT2}}}$$

$$Q_{INT1} = \text{-A} = -\frac{GB}{\omega} = -\frac{1MHz}{10KHz} = -100$$

$$Q_{INT2} \cong -\frac{1}{3}\left|A(j\omega)\right| = \frac{-1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1MHz}{3 \bullet 10KHz} = -33$$

$$Q_P \cong \frac{1}{\frac{1}{10}-.01-.033} = 17.5$$

Note the nonideal integrators cause about a 75% shift in $Q_P$

Note that 3 times as much of the shift is due to the noninverting integrator as is due to the inverting integrator!

Similar effects of the integrators will be seen on other filter structures

# Example:

If $f_0$=10KHz, GB=1MHz, $Q_{NOM}$=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}}$$

$$Q_{INT1} = \text{-A} = -\frac{GB}{\omega} = -\frac{1MHz}{10KHz} = -100$$

$$Q_{INT2} \simeq -\frac{1}{3}\left|A(j\omega)\right| = \frac{-1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1MHz}{3 \bullet 10KHz} = -33$$

$$Q_P \cong \frac{1}{\frac{1}{10} - .01 - .033} = 17.5$$

How can the problem be solved?

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

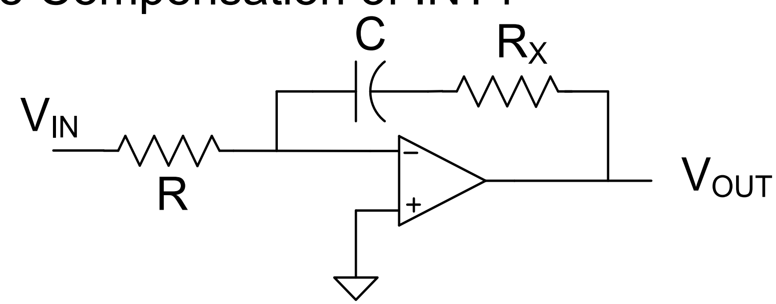
# Example:

If $f_0$=10KHz, GB=1MHz, $Q_{NOM}$=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}}+\frac{1}{Q_{INT1}}+\frac{1}{Q_{INT2}}} \qquad Q_{INT1}=-100 \qquad Q_{INT2}\simeq -33$$

How can the problem be solved?

Phase Compensation of INT1




$$A_V(s)=\frac{-(1+R_XCs)}{RCs+\tau s(1+[R+R_X]Cs)}$$

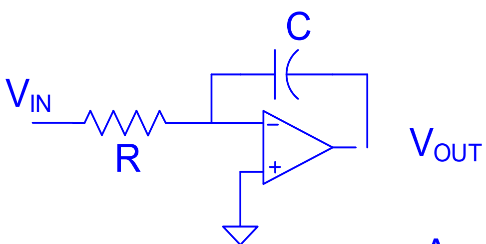
$$Q_{INT1}=\frac{-GB/\omega}{1-GB\bullet CR_X}$$

Pick $R_x$ so that $Q_{INT1}$=33 at $\omega$=1/(RC)

Solving, obtain $CR_x$=4/GB

Useful for hand calibration but not practical for volume production because of variability in components

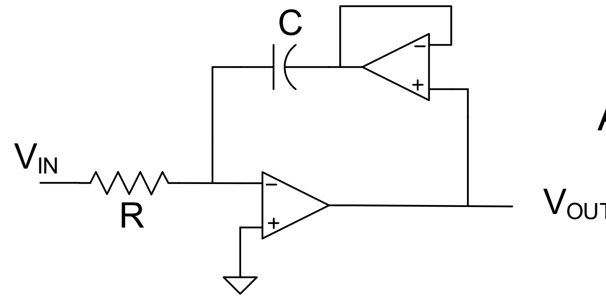
# What are the integrator Q factors for other integrators that have been considered?




$Q_{INT} = -A$

$$A_V(s) = -\frac{1}{RCs}$$

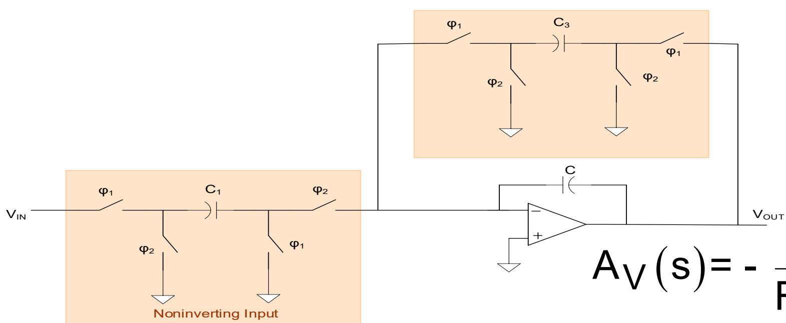
Miller Inverting




$$A_V(s) = -\frac{1}{RCs}$$

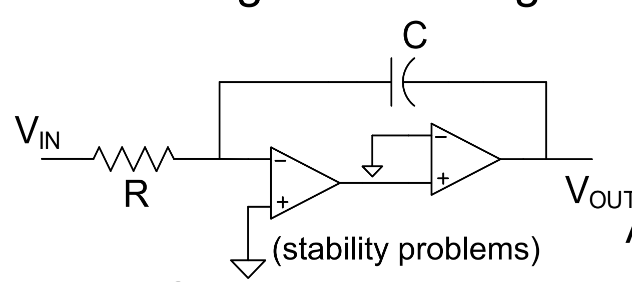
$Q_{INT} = -A^3$

High-Q Inverting



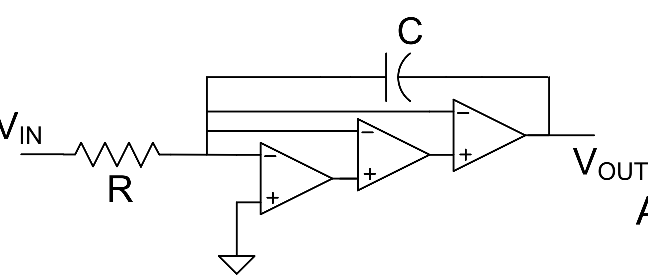

$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



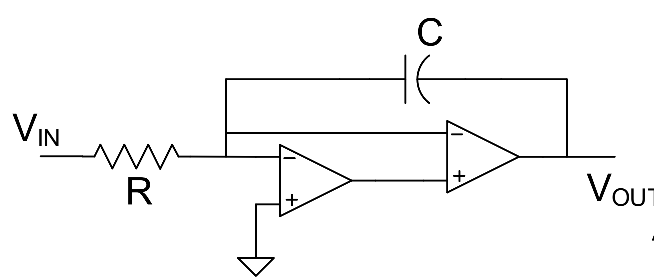

$$A_V(s) = -\frac{1}{RCs}$$

Cascaded Inverting




$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting

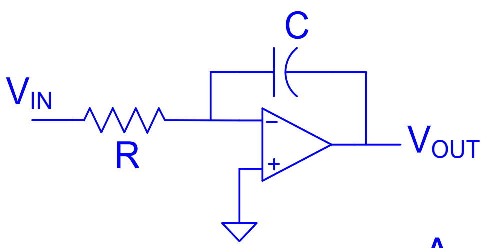



$Q_{INT} = -A^2$

$$A_V(s) = -\frac{1}{RCs}$$

Zero Sensitivity Inverting

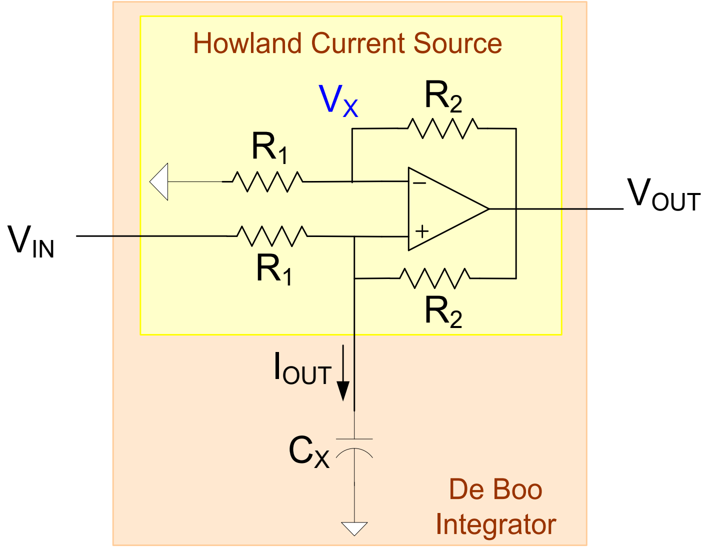
# What are the integrator Q factors for other integrators that have been considered?




Miller Inverting

$$Q_{INT} = -A$$

$$A_V(s) = -\frac{1}{RCs}$$



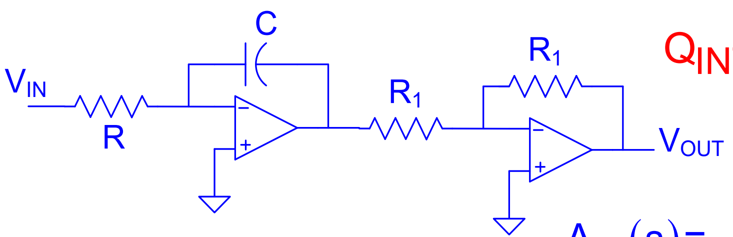

$$A_V(s) = \frac{1+\frac{R_2}{R_1}}{R_1Cs + \tau_1 s\left(1+\frac{R_2}{R_1}\right)\left(1+\frac{R_1}{R_2}+sCR_1\right)}$$

$$Q_{INT} = -\frac{A}{1+\frac{R_2}{R_1}}$$

If $R_1=R_2=R$ $\quad A_V(s) = \frac{2}{RCs}$

$$Q_{INT} = -\frac{A}{2}$$

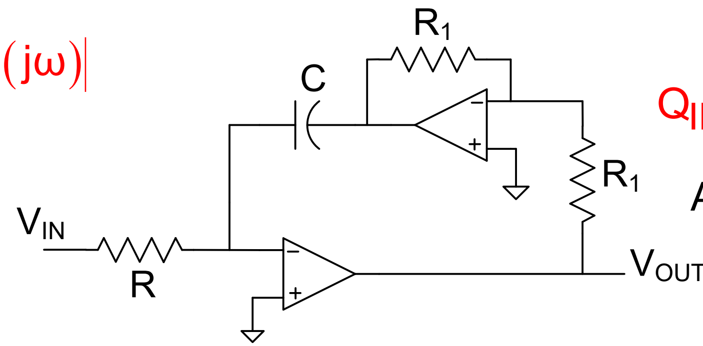
# What are the integrator Q factors for other integrators that have been considered?




$$Q_{INT} \simeq -\frac{1}{3}\left|A(j\omega)\right|$$

$$A_V(s) = \frac{1}{RCs}$$

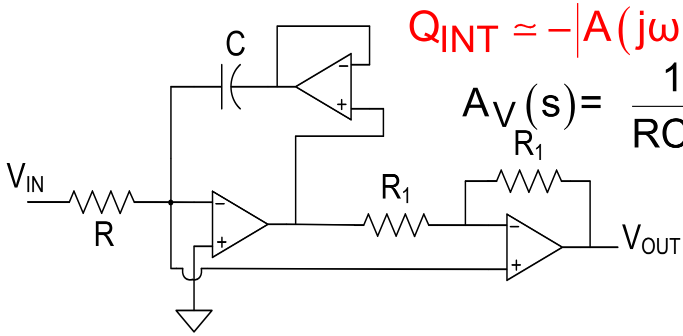
Miller Noninverting




$$Q_{INT} \simeq \left|A(j\omega)\right|$$

$$A_V(s) = \frac{1}{RCs}$$

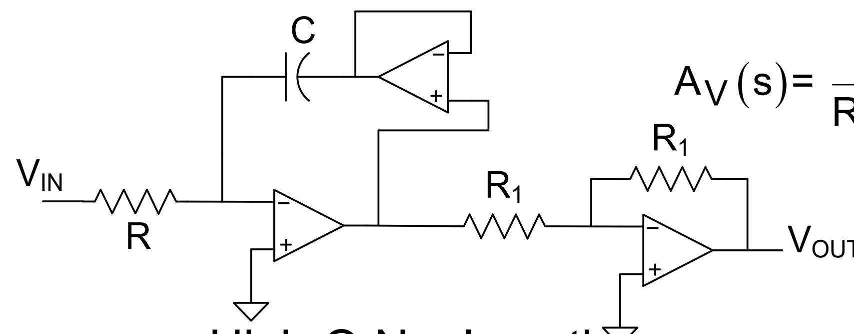
Phase Lead Integrator




$$Q_{INT} \simeq -\left|A(j\omega)\right|^3$$

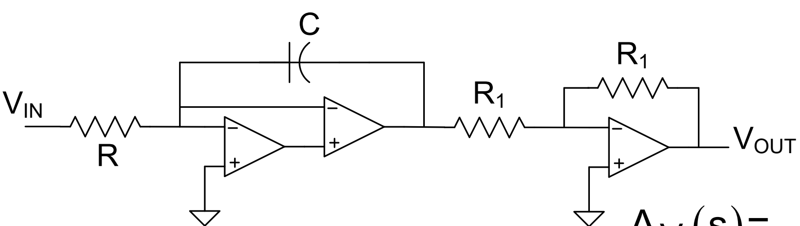
$$A_V(s) = \frac{1}{RCs}$$

Modified High-Q Noninverting



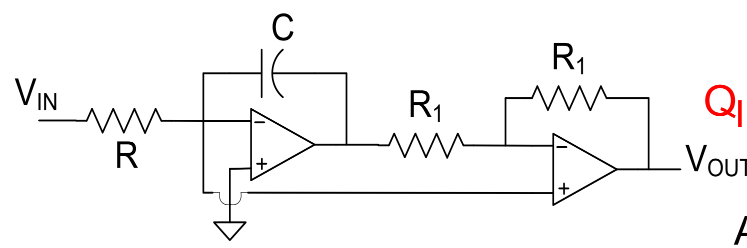

$$A_V(s) = \frac{1}{RCs}$$

High-Q NonInverting




$$A_V(s) = \frac{1}{RCs}$$

Zero Sensitivity Noninverting




$$Q_{INT} \simeq -\left|A(j\omega)\right|$$

$$A_V(s) = \frac{1}{RCs}$$

Modified Miller Noninverting

# Improving Integrator Performance:

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

- These methods all provide some improvements in integrator performance
- But both magnitude and phase of an integrator are important so focusing only on integrator Q factor only may only improve performance to a certain level
- In higher-order integrator-based filters, the linearity in $1/\omega$ of the integrator gain is also important. The integrator magnitude and Q factor at $\omega_0$ ignore the frequency nonlinearity that may occur in the $1/\omega$ dependence
- There is little in the literature on improving the performance of integrated integrators within a basic class. At high frequencies where the active device performance degrades, particularly in finer-feature processes, there may be some benefits that can be derived from architectural modifications along the line of those discussed in this lecture

Stay Safe and Stay Healthy !

End of Lecture 28